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ELECTRIC POTENTIAL ON SOLID SPHERES IN A PLASMA

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Abstract. We derive the general expression for the potential on a solid sphere immersed in a plasma, showing the dependence of the potential on the radius a of the sphere and s of the plasma sheath that develops around the sphere. In the limit where the radius a is much larger than the sheath thickness $s-a$ we recover from this expression the well-known result for the potential on an infinite wall in contact with a plasma. On the other extreme where s is much larger than a, we get the result derived by Spitzer (1941) for the potential on spherical grains in the interstellar plasma. Since the surface of the sphere forms a sink for the charged particles, there is a net drift of the plasma towards the surface. The effect of this drift on the potential is examined. Finally, for very small metallic spheres, an effect leading to a revision of the potential is discussed. This effect consists in a lowering of the potential barrier for the electrons due to the image force. The various effects limiting the potential on spheres are discussed.

1. Introduction

A solid body immersed in a plasma is continually impacted on by ions and electrons. One normally assumes that the ions and electrons hitting the body stick to its surface. They may return to the plasma as neutral atoms, or in some cases the solid body may grow in size with time as a result of condensation of plasma on the surface. We shall assume here that the impinging electrons and the ions have the same probability of sticking to the surface. The analysis is easily modified to take into account the case where these probabilities are different but have a fixed known ratio. Since the thermal velocity of the electrons is much higher than that of the ions, the electron flux to the surface initially far exceeds the ion flux. As a result there is an accumulation of electrons on the surface. The negative potential associated with these electrons provides an attractive force for the ions and a repulsive force for the electrons so that the ion flux increases and the electron flux decreases until an equilibrium is reached where these two fluxes are equal and the potential on the surface has a fixed negative value. This value has been calculated in the astrophysical context by Spitzer (1941, 1968), Wickramasinghe (1967), and others. In laboratory experiments dealing with electrical probes for measuring the physical conditions in a plasma, this potential has long been known as the *floating potential.* In many problems related to plasma-grain systems encountered in astrophysics as well as colloidal plasmas in the laboratory it is useful to know the value of this potential.

In this paper we shall evaluate the equilibrium potential on solid spheres immersed in a plasma following essentially the lines of early work of Langmuir on electrical probes (see e.g. Suits, 1961). We shall show that for a plasma of given density and temperature (i.e. given Debye shielding distance) there are three different domains

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1974Ap&SS..30..135D

of the radius of the sphere in which different approximations for the potential must be used. These domains may be characterized as (a) the thin sheath domain, where the radius a of the sphere is much larger compared to the thickness $s-a$ of the Debye sheath that forms around it ($s=$ the radius of the Debye sheath), (b) the thick sheath domain, where a is much smaller than s and (c) the transition domain, where a is comparable to $s-a$. Finally, in each of these cases, for metallic spheres with radii in the submicron range, we shall discuss an effect which seems to be of some significance. This effect consists in a lowering of the potential barrier that an electron has to overcome in order to reach the surface of the sphere; this lowering results from the image force on the electron when it is close to the metal sphere. Finally, we shall discuss some effects that set an upper limit to the potential for both metallic and dielectric spheres.

We shall assume throughout this paper that the Debye sheaths of the spheres do not intersect one another. The plasma will be assumed collisionless – i.e. the effect of collisions within the Debye sheath will be neglected. This assumption is reasonable in most astrophysical and laboratory plasma-particle systems. The discussion in this paper will further be restricted to the cases where any relative velocity of the sphere with respect to the plasma is much smaller than the ion thermal velocity.

2. The Electric Potential on a Sphere

We shall consider a sphere of radius a immersed in a fully ionized plasma where the atoms are singly ionized. Apart from the effect of collisions, the analysis given here is easily generalized for a partially ionized plasma or a plasma containing multiply ionized atoms, if the electron and ion densities are known. We shall not consider any processes of emission of electrons from the sphere (other than field emission from metal spheres), but such processes will be discussed briefly in Section 6.

Let ϕ_a be the steady electric potential developed on the sphere. The radius s of the plasma sheath around the sphere is approximately given by

$$s \approx a + \lambda_D = a + 6.9\left(\frac{T}{N}\right)^{1/2}, \tag{1}$$

where λ_D is the Debye shielding distance, N is the electron density and T is the plasma temperature (assumed same for electrons and ions) in cgs units. Let q_α be the charge of a plasma particle ($=-e$ for electrons and e for ions) and m_α be its mass ($=m_e$ for electrons and m_i for ions). If u and v are the radial and tangential components of the particle velocity, and the subscripts a and s refer to their values at the surface of the sphere and at the sheath boundary respectively, then for a particle that crosses the sheath and arrives at the surface, the laws of conservation of energy and angular momentum require

$$\tfrac{1}{2}m_\alpha u_s^2 + \tfrac{1}{2}m_\alpha v_s^2 = \tfrac{1}{2}m_\alpha u_a^2 + \tfrac{1}{2}m_\alpha v_a^2 + q_\alpha \phi_a \tag{2}$$

and

$$s v_s = a v_a. \tag{3}$$

1974Ap&SS..30..135D

For a particle which is barely able to arrive at the surface, we put $u_a = 0$ and find from the above two equations a value $V_{s\alpha}$ of v_s given by

$$V_{s\alpha}^2 = \frac{a^2}{s^2 - a^2}\left(u_s^2 - \frac{2q_\alpha \phi_a}{m_\alpha}\right). \tag{4}$$

Clearly, the particles for which v_s exceeds $V_{s\alpha}$ cannot arrive at the surface. Let us assume that at the sheath boundary the electrons and ions have Maxwellian velocity distribution. In our spherical coordinate system this distribution is given by

$$n_\alpha(u, v)\,\mathrm{d}u\,\mathrm{d}v = 2\pi N\left(\frac{m_\alpha}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m_\alpha}{2kT}(u^2 + v^2)\right] v\,\mathrm{d}u\,\mathrm{d}v; \tag{5}$$

and, hence, the total flux of ions and electrons to the surface is given by

$$\Gamma_i = 4\pi s^2 \int_0^\infty \int_0^{V_{si}} n_i(u_s, v_s)\, u_s\,\mathrm{d}v_s\,\mathrm{d}u_s \tag{6}$$

and

$$\Gamma_e = 4\pi s^2 \int_{U_{se}}^\infty \int_0^{V_{se}} n_e(u_s, v_s)\, u_s\,\mathrm{d}v_s\,\mathrm{d}u_s, \tag{7}$$

where U_{se} is the lowest radial electron velocity for which it can overcome the repulsive force due to the negative potential on the surface and is given by

$$\tfrac{1}{2} m_e U_{se}^2 = -e\phi_a. \tag{8}$$

Evaluation of the above integrals gives

$$\Gamma_i = 4\pi s^2 N\left(\frac{kT}{2\pi m_i}\right)^{1/2} \left\{1 - \frac{s^2 - a^2}{s^2} \exp\left[\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT}\right]\right\}, \tag{9}$$

$$\Gamma_e = 4\pi a^2 N\left(\frac{kT}{2\pi m_e}\right)^{1/2} \exp\left[\frac{e\phi_a}{kT}\right]. \tag{10}$$

The potential ϕ_a is now given by the condition

$$\Gamma_i = \Gamma_e \tag{11}$$

or

$$\exp\left[\frac{e\phi_a}{kT}\right] = \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{s}{a}\right)^2 \left\{1 - \frac{s^2 - a^2}{s^2} \exp\left[\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT}\right]\right\}, \tag{12}$$

If the electrons and the ions in the plasma have different temperatures T_e and T_i, then the above result becomes

$$\exp\left[\frac{e\phi_a}{kT_e}\right] = \left(\frac{m_e T_i}{m_i T_e}\right)^{1/2} \left(\frac{s}{a}\right)^2 \left\{1 - \frac{s^2 - a^2}{s^2} \exp\left[\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT_i}\right]\right\}. \tag{13}$$

However we shall not discuss this case any further. We now observe the following two limiting cases of Equation (12):

1974Ap&SS..30..135D

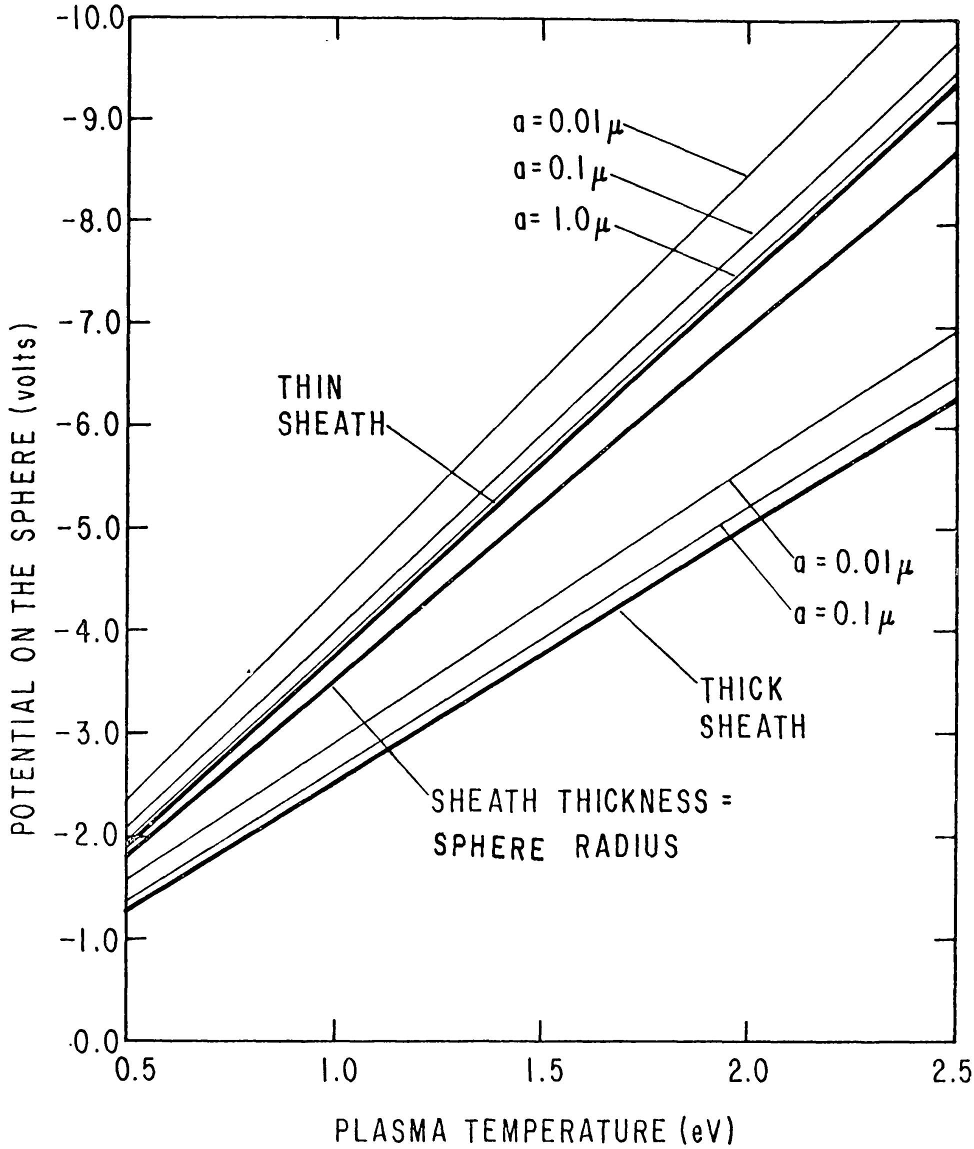

Fig. 1. The potential of solid spheres in a hydrogen plasma as a function of the plasma temperature. The thick curve at the bottom corresponds to the thick sheath case with no image force (Equation (16)), and the thin curves immediately above it correspond to the thick sheath case including the effect of image force (Equation (39)) for various values of the radius of the sphere. The thick curve at the top is for the thin sheath case with no image force (Equation (14)) and the thin curves above it are for the same case including the image force (Equation (38)). The thick curve in the middle is for an intermediate case (Equation (12)) where the sheath thickness $s-a$ equals the radius a of the sphere. Note that the thick curves apply to both metallic and dielectric spheres while the thin curves apply only to metallic spheres. The results presented in this figure are subject to the constraint discussed in Section 5.

The thin sheath domain: If the conditions in the plasma are such that the sheath thickness $s-a$ is much smaller than the radius a of the sphere, then Equation (12) reduces to (note that ϕ_a is negative)

$$\exp\left[\frac{e\phi_a}{kT}\right]=\left(\frac{m_e}{m_i}\right)^{1/2}. \tag{14}$$

1974Ap&SS..30..135D

For a fully ionized hydrogen plasma, this result leads to

$$\frac{e\phi_a}{kT} = -\tfrac{1}{2}\ln\left(\frac{m_i}{m_e}\right) \approx -3.76. \tag{15}$$

Note that for the plasma in the sheath, the surface of the sphere in this case behaves like a plane wall. Hence the result in Equation (14) is the same as that for the potential on an infinite wall in contact with a plasma. In Figure 1 we have plotted the potential as a function of the plasma temperature for the case of the hydrogen plasma.

The thick sheath domain: On the other extreme if the conditions in the plasma are such that the sheath radius s is very large compared to the sphere radius a, then Equation (12) reduces to

$$\exp\left[\frac{e\phi_a}{kT}\right] = \left(\frac{m_e}{m_i}\right)^{1/2}\left[1 - \frac{e\phi_a}{kT}\right], \tag{16}$$

which for a fully ionized hydrogen plasma yields

$$\frac{e\phi_a}{kT} \approx -2.51. \tag{17}$$

This result is the same as that derived by Spitzer (1941, 1968) in the astrophysical context and probably applies to most situations of astrophysical interest, e.g. micron size grains in the tenuous interstellar plasma. In Figure 1 we have also plotted Equation (17).

The transition domain: In the cases where s is comparable to a however, we must use Equation (12) without making any approximations. In Figure 1 we have plotted a particular case where $s = 2a$.

3. The Effect of Plasma Drift

One effect which seems to be important in discussing the potential on solid surfaces in contact with a plasma is the following: Since the plasma particles neutralize on the solid surface, this surface is in effect a sink for the plasma. This means that there is a net drift of the plasma towards the surface.

In order to examine this effect, suppose u_D is the drift velocity of the plasma across the boundary of the Debye sheath. Then the net flux of ions across the sheath boundary to the surface of the sphere is given by

$$\Gamma_{i,\mathrm{D}} = 4\pi s^2 \int_{-u_D}^{\infty} \int_{0}^{V_{si}} n_i(u_s, v_s)\,(u_s + u_\mathrm{D})\,\mathrm{d}v_s\,\mathrm{d}u_s. \tag{18}$$

However, the electrons with insufficient velocities cannot reach the surface of the sphere and hence the electron flux is still given by Equation (7). Equating the electron and the ion fluxes, we now find the following relation for the potential ϕ_a

$$\exp\left[\frac{e\phi_a}{kT}\right] = \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{s}{a}\right)^2\left[\{e^{-p^2} + \pi^{1/2}p[1 + \operatorname{erf}(p)]\}\right.$$
$$\left. - \frac{s^2 - a^2}{s^2}\exp\left[\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT}\right]\{e^{-q^2} + \pi^{1/2}q[1 + \operatorname{erf}(q)]\}\right], \tag{19}$$

where

$$p^2 = \frac{m_i}{2kT}u_{\mathrm{D}}^2, \tag{20}$$

$$q^2 = \frac{s^2}{s^2 - a^2}\frac{m_i}{2kT}u_{\mathrm{D}}^2. \tag{21}$$

There is no way to *calculate* the drift velocity u_{D}, but we can *estimate* u_{D} by requiring that the flux of ions per unit area to the surface of the sphere evaluated at the surface equals the flux of ions per unit area towards the surface evaluated at a point outside the Debye sheath. The latter flux is simply Nu_{D}. The former flux is found by dividing the expression in Equation (9) by $4\pi a^2$, and replacing the ion density N by its value N_a at the surface. This gives the flux γ_i of ions per unit area at the surface as

$$\gamma_i = N_a\left(\frac{kT}{2\pi m_i}\right)^{1/2}\left(\frac{s}{a}\right)^2\left\{1 - \frac{s^2 - a^2}{s^2}\exp\left(\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT}\right)\right\}, \tag{22}$$

where N_a is given by the Boltzmann distribution

$$N_a = N\exp\left[-\frac{e\phi_a}{kT}\right]. \tag{23}$$

Equating the flux given by Equation (22) to Nu_{D} we find a first order approximation for u_{D} to assume the form

$$u_{\mathrm{D}} = \left(\frac{kT}{2\pi m_i}\right)^{1/2}\exp\left[-\frac{e\phi_a}{kT}\right]\left(\frac{s}{a}\right)^2\left\{1 - \frac{s^2 - a^2}{s^2}\exp\left[\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT}\right]\right\}. \tag{24}$$

If we evaluate u_{D} for the electrons in a similar way and remember that γ_i and γ_e must be equal, we find that u_{D} for the electrons has the same value as u_{D} for the ions. Thus, electrons and ions drift together with a common velocity towards the surface of the sphere. In the *thin sheath* case, u_{D} becomes

$$u_{\mathrm{D}} = \left[\frac{kT}{2\pi m_i}\right]^{1/2}\exp\left(-\frac{e\phi_a}{kT}\right); \tag{25}$$

and in the *thick sheath* case, u_{D} becomes

$$u_{\mathrm{D}} = \left(\frac{kT}{2\pi m_i}\right)^{1/2}\exp\left[-\frac{e\phi_a}{kT}\right]\left[1 - \frac{e\phi_a}{kT}\right]. \tag{26}$$

Taking the thin and the thick sheath limits of Equation (19), we now find that the

1974Ap&SS..30..135D

potential ϕ_a in the *thin sheath* case is given by

$$\exp\left[\frac{e\phi_a}{kT}\right]=\left(\frac{m_e}{m_i}\right)^{1/4}, \tag{27}$$

which for a hydrogen plasma becomes

$$\frac{e\phi_a}{kT}=-\tfrac{1}{4}\ln\left(\frac{m_i}{m_e}\right)\approx-1.88. \tag{28}$$

In the *thick sheath* case, the potential ϕ_a is given by

$$\exp\left[\frac{e\phi_a}{kT}\right]=\left(\frac{m_e}{m_i}\right)^{1/4}\left[1-\frac{e\phi_a}{kT}\right] \tag{29}$$

which for a hydrogen plasma becomes

$$\frac{e\phi_a}{kT}\approx\sim 1.11. \tag{30}$$

These results are significantly different from the earlier results, illustrating the importance of the drift velocity. The results in Equations (14) and (16) thus represent *upper limits* to the (negative) potential on the sphere.

4. Submicron Size Metal Spheres: The Effect of Image Force

If the sphere has a charge $-Ze$ on it, then in a spherical coordinate system with origin at the center of the sphere, the potential energy of an electron at points exterior to the sphere is given by

$$-e\phi_r=\frac{Ze^2}{r}. \tag{31}$$

This potential is shown by the dotted curve in Figure 2(a). The assumption of the Coulomb potential rather than the Debye potential is justified by the fact that very close to the sphere (which is the region of our interest), the difference between the two results is very small; at the same time, the assumption of the Coulomb potential simplifies the analysis greatly.

An electron near the sphere is also attracted towards it due to the electron's own image in the sphere. The corresponding potential energy is

$$e\phi_i=-\frac{e^2a}{2\left(r^2-a^2\right)}+\frac{e^2a}{2r^2}. \tag{32}$$

This is shown by the broken line in Figure 2(a). The full line in this figure shows the sum of these two energies

$$-e\phi=\frac{Ze^2}{r}-\frac{e^2a}{2\left(r^2-a^2\right)}+\frac{e^2a}{2r^2}. \tag{33}$$

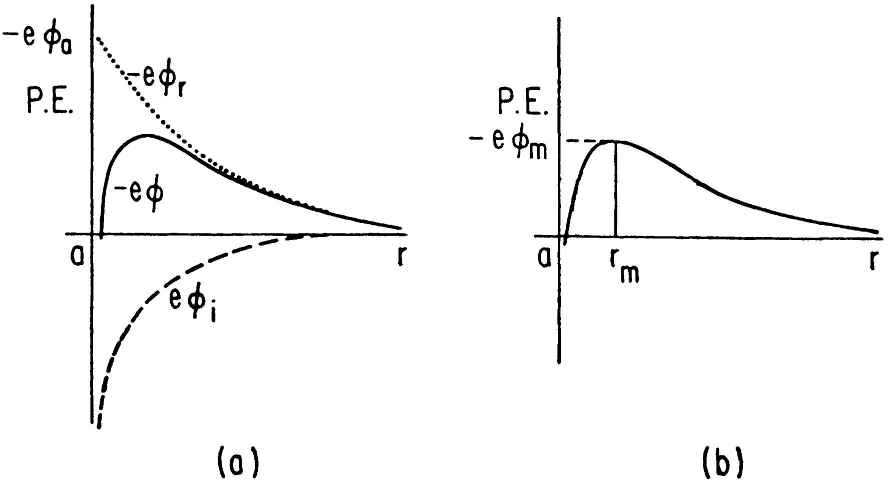

Fig. 2. (a) Dotted curve: Potential energy of an electron near a sphere of radius a due to a charge $-Ze$ on the sphere. Broken curve: Potential energy of an electron near a metal sphere due to its own image force. Solid curve: The net potential energy of an electron near a metal sphere. (b) The maximum potential ϕ_m and the position r_m of the maxima.

This curve therefore gives us the potential barrier that an electron would have to surmount in order to reach the surface of the sphere. It is not necessary for our discussion to match the curve to a specific value at the surface of the sphere.

The position r_m of the barrier maxima (Figure 2(b)) is given by the solution of the equation $d\phi/dr = 0$. This equation can be solved numerically to get values of r_m for different values of Z. However, for our purpose it is very convenient to have an analytical expression for r_m as a function of Z. Such an expression was found by guessing and trial as

$$r_m = a\left[1 - \frac{0.17}{Z^{1/2}}\right]\left(\frac{1}{2Z}\right)^{1/2}\left[1 + (1 + 2Z)^{1/2}\right]. \tag{34}$$

In Figure 3 this analytical solution is found to be indistinguishable from the numerical solution. The agreement continues to be good for large values of Z. Note that Equation (34) has the correct behavior in that r_m approaches a for very large values of Z. The energy $-e\phi_m$ of the barrier maxima is now given by

$$-e\phi_m = \frac{Ze^2}{r_m} - \frac{e^2 a}{2(r_m^2 - a^2)} + \frac{e^2 a}{2r_m^2}. \tag{35}$$

The effect of the image force of lowering the energy barrier for incoming electrons from $-e\phi_a$ to $-e\phi_m$ is easily included in our analysis by replacing Equation (8) by

$$\tfrac{1}{2}m_e U_{se}^2 = -e\phi_m; \tag{36}$$

so that Equation (12) for the surface potential ϕ_a now becomes

$$\exp\left[\frac{e\phi_m}{kT}\right] = \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{s}{a}\right)^2\left\{1 - \frac{s^2 - a^2}{s^2}\exp\left[\frac{a^2}{s^2 - a^2}\frac{e\phi_a}{kT}\right]\right\}, \tag{37}$$

where the relationship between ϕ_m and ϕ_a is given by Equations (34), (35) and the

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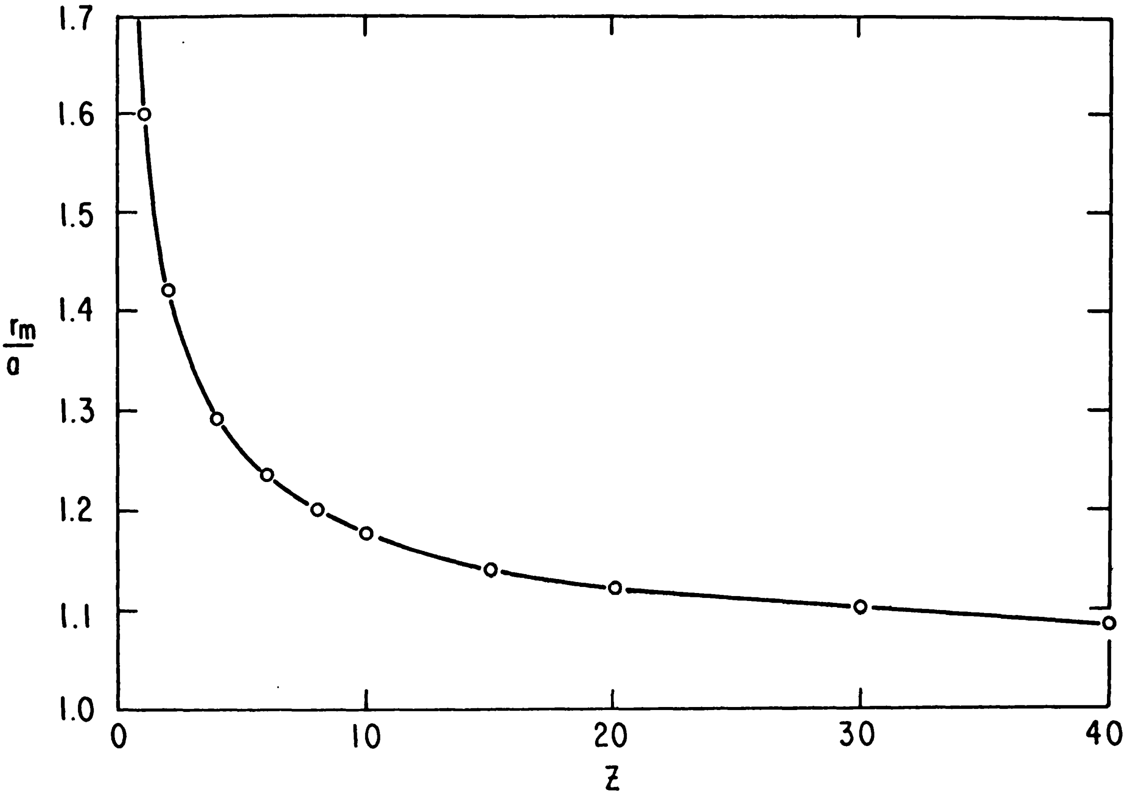

Fig. 3. The position r_m of the maxima of the potential as a function of the number of electrons Z on the surface of the sphere. The solid curve represents the approximate solution (Equation (34)). The circles represent numerical solutions.

relation $\phi_a = -Ze/a$. Equation (14) for the thin sheath case now becomes

$$\exp\left[\frac{e\phi_m}{kT}\right] = \left(\frac{m_e}{m_i}\right)^{1/2}; \tag{38}$$

and Equation (16) for the thick sheath case reduces to

$$\exp\left[\frac{e\phi_m}{kT}\right] = \left(\frac{m_e}{m_i}\right)^{1/2}\left[1 - \frac{e\phi_a}{kT}\right]. \tag{39}$$

In Figure 1 we have presented some results for the potential of the sphere as a function of plasma temperature including the image effect, for both the thin and thick sheath limits. When the image effect is included, the potential becomes a function of the radius of the sphere and the effect is unimportant for radii larger than $a = 1\ \mu$. In the thin sheath case, an additional condition for the image effect to be operative is that s must be larger than r_m. Note that since the image force permits a larger electron flux to the surface, the potential becomes larger when this effect is included.

In Figure 4 we have plotted the potential on a sphere as a function of the ratio s/a with $s = 1$ m. This figure shows the various domains of the quantity s/a in which different approximations for the potential should be used.

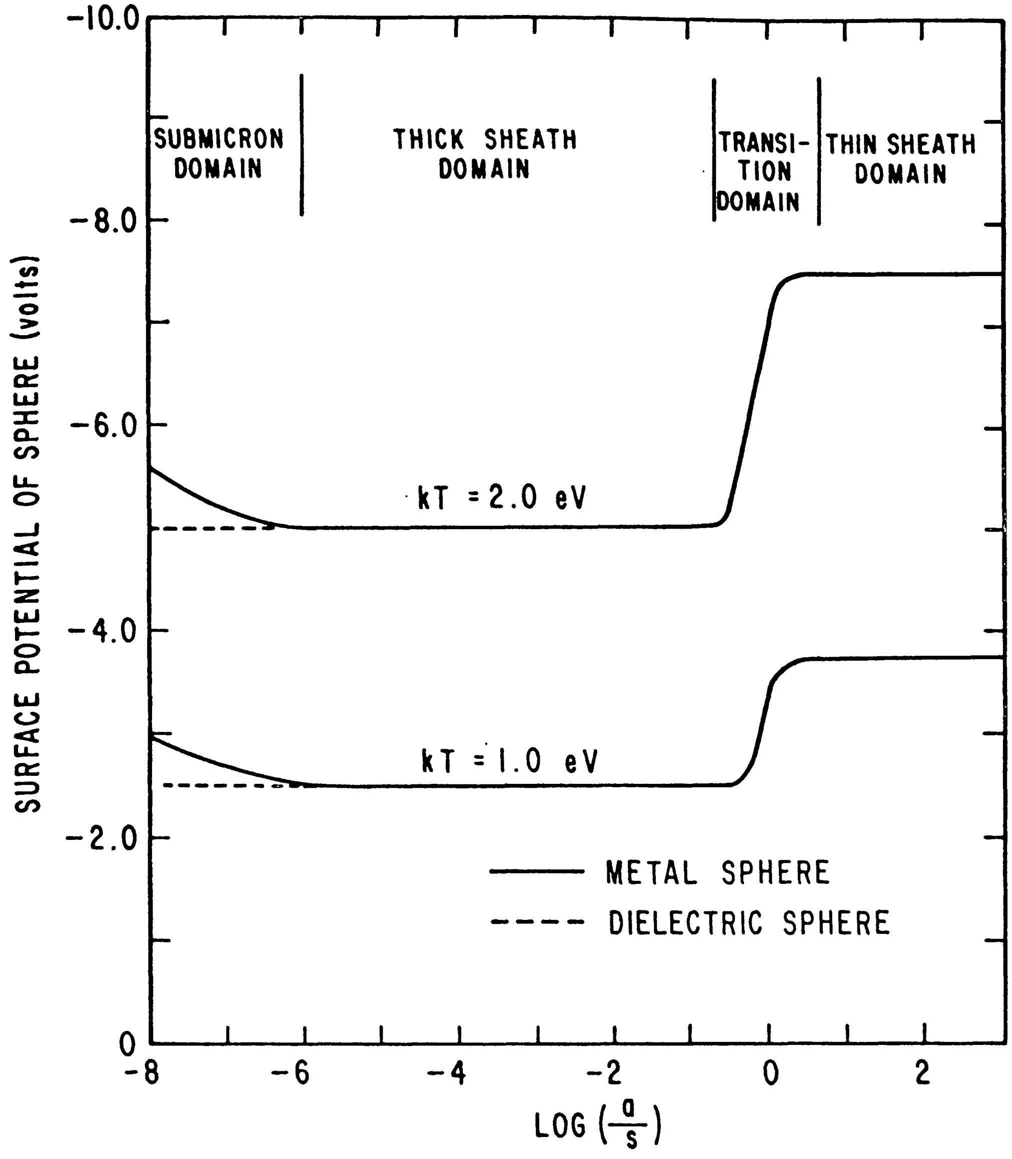

Fig. 4. The potential of solid spheres in a hydrogen plasma as a function of the sphere radius in units of s, the radius of the Debye sheath which has been taken to be 1 m.

5. Limitation of the Potential

Our discussion so far indicates that the potential on a solid sphere in a plasma increases monotonically with increasing plasma temperature. However, this increase cannot take place indefinitely. In the case of dielectric spheres, a limit to the potential is set by either the dielectric breakdown phenomenon or a rupture of the sphere due to electrical forces, whichever occurs at a lower potential. If E_B is the breakdown electric field for the material of the sphere, then the maximum potential that the sphere can sustain is

$$\phi_a(\max) \approx aE_B. \tag{40}$$

The criterion for rupture is hard to establish, but we can obtain a crude estimate in the following way. There is a uniform pressure $p = \phi_a^2/4\pi a^2$ over the surface of the sphere due to electrical forces. This pressure induces a tensile stress p throughout the body of the sphere. The criterion for a rupture of the sphere to occur is that p must exceed the tensile strength p_t of the material of the sphere. From this consideration, the maximum potential that the sphere can sustain without breaking up is given by

$$\phi_a(\max) \approx 600\, a(\pi p_t)^{1/2}, \tag{41}$$

where ϕ_a is in volts and a and p_t are in cgs units. As an example, for glass $\phi_a(\max)$ is about 13 a_μ V according to Equation (40) and about 3×10^3 a_μ V according to Equation (41), where a_μ is the radius of the sphere in microns.

For metal spheres, however, the limit is set by the field emission of electrons under the influence of the electric field

$$E = \frac{\phi_a}{a}. \tag{42}$$

The flux of field-emitted electrons from a metal surface is given by the Fowler-Nordheim equation (see e.g. Gomer, 1961)

$$\Gamma_{\text{fem}} = 4\pi a^2 \frac{(\beta/w)^{1/2}}{\xi^2(\beta + w)} E^2 \exp\left[58.9 - 6.8 \times 10^7\, w^{3/2} \frac{\xi}{E}\right], \tag{43}$$

where

$$\xi = (1 - y)^{1/2},$$
$$y = 3.8 \times 10^{-4}\, E^{1/2}/w;$$

and β and w are the Fermi energy and the work function for the metal in eV and E is in volts per cm. The Fowler-Nordheim equation includes the effect of the image force on the emitted electron. The steady potential on the sphere including the effect of field emission is given by the equation

$$\Gamma_i = \Gamma_e - \Gamma_{\text{fem}}/\chi, \tag{44}$$

where χ is the sticking probability of electrons and ions and Γ_i and Γ_e are given by Equations (9) and (10). Note that unlike the solution of Equation (11), the solution of Equation (44) depends on the value of N, the density of the ions or the electrons.

In Figure 5 we have plotted the solution of Equation (44) for a sphere of radius 0.01 μ and $\beta = 11.6$ eV and $w = 3.0$ (values for aluminum) for the thin sheath case, assuming $\chi = 1$. Obviously, the effect of field emission is to put an upper limit to the potential on the sphere. This figure also shows the dependence of the solution on the plasma density N. To find the upper limit of the potential for a sphere of a different size, we have simply to multiply the upper limit shown in Figure 5 by $10^2\, a_\mu$. The upper limits found here do not change appreciably if we include in the electron flux Γ_e in Equation (44) the effect of the image force as we have done in Section 4.

In the thick sheath case or in a general case, the upper limit of the voltage, in addition to being a function of a and N, becomes dependent on the radius s of the Debye sphere. This potential is easily calculated from Equation (44).

1974Ap&SS..30..135D

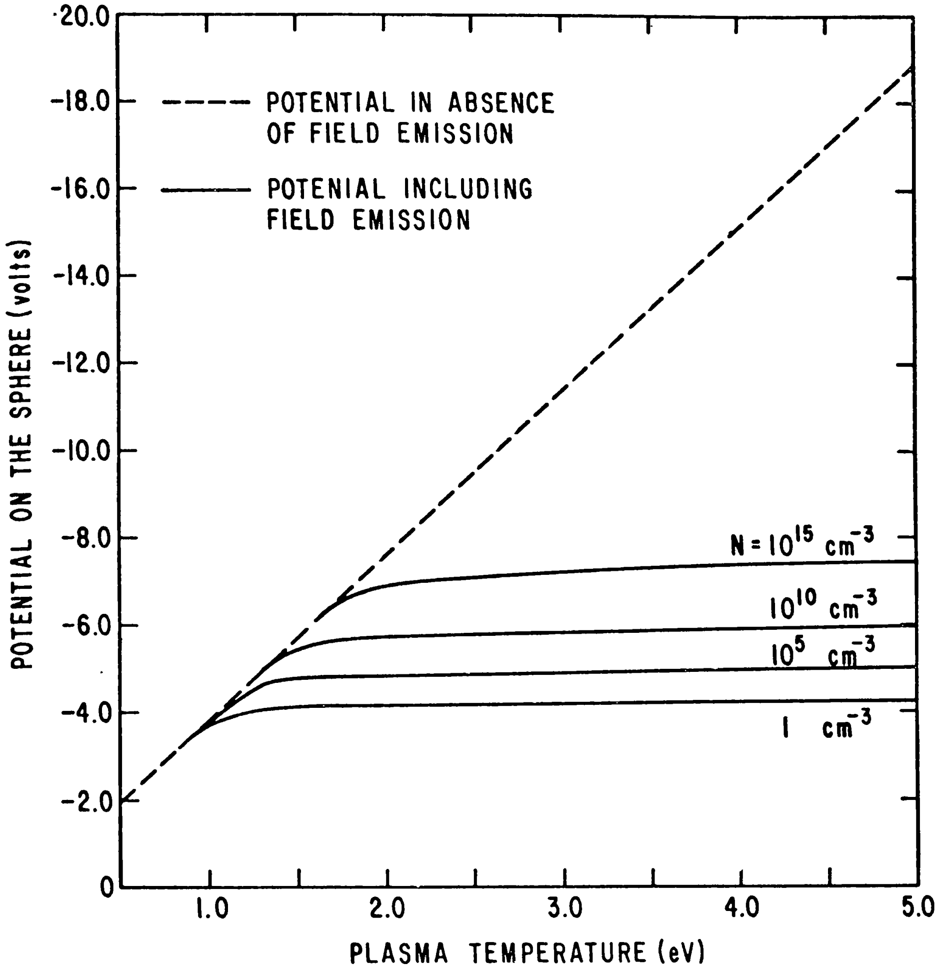

Fig. 5. Limitation of potential due to field emission of electrons. The broken line represents the potential for the thin sheath case without including the effect of field emission (Equation (14)) in the case of hydrogen plasma. The solid lines represent this potential in the same case including the effect of field emission (Equation (44)) for a sphere of radius 0.01 μ and for various plasma densities N, assuming the sticking probability χ of the ions and the electrons to be equal to 1. However, note that the result for $N = 10^{10}$ cm^{-3} and $\chi = 1$ for instance would be the same as the result for $N = 10^{15}$ cm^{-3} and $\chi = 10^{-5}$. This gives us an idea of the dependence of the result on χ.

The Fowler-Nordheim equation estimates the flux of field-emitted electrons from a plane surface. According to Dubey (1970), the flux from a spherical surface may be somewhat higher because whereas the classical criterion for an electron to escape from a plane surface is that its normal component of energy must exceed the surface barrier, the classical condition for escape from a spherical surface is that its total energy must exceed the surface barrier. If this effect is taken into account, the upper limit to the potential will be somewhat lower than the values we have found here.

When the incident ion energies are very high, in the range of several hundreds of eV, the effect of secondary emission of electrons becomes important and should be included in the analysis. These energies correspond to plasma temperatures in excess of $\sim 10^6$ K. Such high temperatures are not usually encountered in astrophysical or laboratory plasma-particle systems.

6. Emission of Electrons

We have refrained in this paper from including in our analysis the various processes of emission of electrons from the surface of the sphere. Such processes have been discussed in detail in literature and can be easily included in our analysis once the specific nature of the problem is known. For instance, in discussing photoemission of electrons, we need a knowledge of the photon field in the vicinity of the sphere as well as the electrical properties of the material of the sphere (see e.g., Spitzer, 1941). Once the efflux of electrons due to one or more of the various emission processes is known, this can be subtracted from the electron influx in Equation (11). The solution of the resulting equation would give us the potential in this case.

The reader is referred to an excellent review of electron emission processes from solid surfaces by Sodha and Guha (1971). Because of the existence of this review, a discussion of the emission processes in this paper seems redundant.

7. Remarks

We have shown that the potential on a solid sphere in a plasma depends on the relative dimensions of the radius of the sphere and the Debye shielding distance in the plasma. We have further shown that it is important to include in the calculation of the potential the effect of a net drift of the plasma towards the sphere. For submicron size metallic grains, we have suggested that the effect of the image force on the electrons near the sphere may result in a revision of the potential. Finally, we have discussed some effects that limit the potential on the spheres so that the potential cannot increase indefinitely with increasing plasma temperature. We have not included in our analysis the various processes of emission of electrons, but they are easily included in a specific problem once the exact nature of the problem is known.

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